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A win-win supply chain solution using project contracts with bargaining games

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ABSTRACT

For product supply chains, contractual relationships that provide win-win outcomes between the supply chain members, have been found to offer optimum results. However, for bargaining situations where time/cost is the source of the uncertainty, i.e. projects, there is limited knowledge available on how contracts can be used to establish win-win relations. This paper investigates whether cost-sharing project contracts can establish a win-win solution in project supply chains where the project manager is risk-neutral and the contractor is risk-averse. The paper examines how the theory can be extended beyond the symmetrical normal distributions to asymmetrical beta and gamma distributions that are more appropriate, and so more often used, for project completion times. Besides using the Nash bargaining approach for analyzing the bargaining process, the paper also analyzes the bargaining problems using the Kalai-Smorodinsky and Utilitarian approaches to bargaining. It was found that the solutions from cost-plus contracts dominate any other form of cost-sharing contract, and so they provide a win-win solution for both members of the supply chain for the cases of Nash and Kalai-Smorodinsky bargaining. However, this is not the case for Utilitarian bargaining. A numerical exercise was conducted to investigate the results and implications of how the models would work in practice. The research shows that from a theoretical perspective, cost-plus contracts are the optimal bargaining solution not only when using a normal distribution, but also when using more appropriate asymmetrical distributions. This optimality is robust for the Nash and Kalai-Smorodinsky bargaining approaches, but not for the Utilitarian approach whose sensitivity to noise makes it an inappropriate choice here.

1. Introduction

This paper analyzes the problem of setting an appropriate contract price between a project manager and a contractor for carrying out a project whose cost is uncertain. Cost uncertainty is a major issue that projects encounter. Numerous examples exist of large projects costing substantially more than their original estimates, e.g. the refurbishment of Wembley stadium, and Denver airport baggage handling system installation project [36]. Misaligned goals and objectives between the members of the supply chain are often cited as one of the main reasons behind this e.g. the case of cost overrun in the Denver airport baggage handling system installation project [36]. Despite these problems, little is known about the tools and techniques that could help organizations to avoid these problems and achieve a win-win solution by aligning the individual goals and objectives with the overall goals and objectives when projects have greater uncertainty [28]. Some authors have attempted to address these challenges through the use of project contracts, but these have been limited to the context they used such as a take it or leave it situation [28] or the optimization was considered for one organization only [4]. Hence, this research investigates if win-win coordination can be achieved when the outcomes are negotiated between a project manager and a contractor faced with cost uncertainty.

Aligning the goals and objectives of the members of the supply chain can reduce the impact of supply chain problems such as information distortion across the supply chain [10] and double marginalization [31]. For product supply chains, numerous papers have addressed these issues. However, these concepts have received much less attention in project settings. A few models have been described that are based on take it or leave it situations like the counterpart problem in manufacturing supply chains [5,28,33]. When the supply chain members negotiate in bargaining situations, the tools required to analyze the process are different in comparison to those that are used in take it or leave it situations. Thus, the existing models for take it or leave it situations may not work correctly in bargaining situations. Interestingly,

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bargaining in a project setting in practice was considered in the study of Bajari et al. [3]. The authors discussed the negotiated contracts in the North California building construction sector during 1995–2000. More recent instances include the re-negotiated building contracts in Dubai post the economic downturn in 2008 [7]. Despite these reports, bargaining situations have received limited attention in the existing models in the literature. Lippman et al. [33] developed a bargaining model using the Nash bargaining approach coupled with a normally distributed cost function, but this is limited by the selection of the cost function and the bargaining approach. Unfortunately, normal distributions are not usually a good model for project cost functions as the cost functions are normally asymmetric with high Skewness and Kurtosis values [2]. Moreover, the use of normal distributions may require certain assumptions that are not applicable in practice e.g. it is unbounded on both sides of the peak and has a non-zero probability of having a negative cost. In fact, the use of bounded distributions is more appropriate in project settings due to the ease of combining with the widely used tool PERT, a tool recommended by the Project Management Book of Practice [39]. The beta distribution has been found to be the most commonly used distribution in this context [17,39,43]. However, there are situations with greater uncertainty around the maximum possible completion cost, where a better distribution with no upper bound on the right tail would be more appropriate. There is a considerable academic debate on the actual nature of these distributions, partly because they are context dependent. Despite all these debates, the project manager can make an estimate of the probability distributions from past experience [17]. Hence, to cover these situations, we used the beta and gamma distributions as a representative of the skewed distributions for the cost functions. The beta distributed cost is representative of bounded distributions (both tails) and the gamma is representative of unbounded distributions (for the right tail). We used cost-based contracts that can be flexibly converted to various contract types including fixed-price and cost-plus. These types of contracts are commonly used as mentioned by the Joint Contracts Tribunal Ltd., UK [26]. Furthermore, we present some recent examples of the use of these cost-based contracts from real-life projects in the discussion section. Additionally, although the Nash bargaining model is widely used, there are equally valid alternatives to it. Hence this research investigates and answers the research questions:

If win-win coordination can be achieved when the outcomes are negotiated between a project manager and a supplier using more appropriate project cost functions? and whether the results depend on the bargaining approach used with three different bargaining approaches (Nash, Kalai-Smorodinsky and Utilitarian) being analyzed?

Our results for the case of a risk-neutral project manager and a risk-averse contractor support the findings of Lippman et al. [33] for Nash and Kalai-Smorodinsky bargaining, but there are some differences when using the Utilitarian approach. However, when both the members of the supply chain have some degree of risk-aversion, we found some different results as partial cost-sharing was an optimal solution.

The rest of this paper is organized as follows: the next section provides a brief literature review; and then descriptions of the basic model are presented for the three different bargaining approaches: Nash Bargaining, Kalai-Smorodinsky Bargaining, and Utilitarian Bargaining. This is followed by the numerical analysis and results. Finally, we discuss the implications of the proposed model along with some concluding remarks.

2. Literature review

For many situations, cooperative bargaining approaches have been found to be beneficial for all the parties involved in the negotiation by offering them win-win outcomes. A recent example is Fan et al. [13] who consider an energy-hub based integrated energy system. However, despite its proven benefits, the application of bargaining models in supply chain management (particularly with respect to supply chain coordination) has received relatively less attention in comparison with other game-theoretic approaches [38]. Among the few papers on bargaining approaches in the supply chain literature, Nash bargaining has been found to be the most commonly used tool. This has been used in the literature to address several supply chain issues such as coordination and benefit sharing in a three echelon distribution channel with a deteriorating product [38], optimization of inventory [16,45], co-operative advertising [24], cost-sharing [45], and profit-sharing [30,37,44]. One of the limitations of these papers is the lack of focus on the risk preference of the members of the supply chain as the members have been assumed to be risk-neutral. However, authors including Abad [1], Huang [23], Gan et al. [14], He and Zhao [20], and Huang and Li [24] used a Nash bargaining approach to propose optimal solutions with differential risk preferences for the members of the supply chain. As an extension to Nash bargaining, authors including Hezarkhani and Kubiak [21], Lin et al. [32], Zheng and Negenborn [47], and Modak and Kelle [34] used generalized Nash bargaining that takes into account the differential bargaining power of the supply chain members. However, a key aspect of these supply chains is that the product's demand is the decision variable. Other notable uses of bargaining models in supply chains include the Nash Bargaining like solution for the division of the uncertain future profits [15], inventory model for a two-echelon supply chain with a declining price-sensitive demand [35], the Rubinstein bargaining model [46] and a few other non-zero sum bargaining algorithms [41,42]. Again, the difference between these supply chains and this paper is that these are supply chains with the product's demand as the decision variable.

In contrast to the supply chains mentioned in the last paragraph, the application of bargaining concepts in a project setting is very limited. To the best knowledge of the authors of the present research, only the models proposed by Lippman et al. [33] have considered bargaining games between the members of a project-based supply chain. However, these models are limited by assuming that the statistical distribution of the cost variable is a normal distribution. The restrictive nature of this assumption was discussed in the Introduction. Thus, the existing model proposed by Lippman et al. [33] might not work correctly with a cost function distributed as a non-normal continuous distribution. Motivated by this problem in practice and the limitations in the literature, this research has examined if the model proposed by Lippman et al. [33] can be extended with gamma and beta distributed cost functions. Moreover, the models by Lippman et al. [33] are only based on Nash's bargaining approach. Hence, it would be valuable to see whether the results are consistent with other bargaining approaches. This is important as although the Nash bargaining approach is widely employed in the economics literature, alternative bargaining models can be regarded as equally valid. To compare the results of bargaining across different bargaining approaches, we derived models using Kalai-Smorodinsky and Utilitarian approaches alongside the Nash bargaining approach. Additionally, the literature mostly considers one member of the supply chain as risk-averse (usually the contractor) and the other as risk-neutral. In practice, there could be some degree of risk aversion for both the members of the supply chain. We analyzed this further using numerical examples.

3. Problem description

A dyadic supply chain with one project manager and one contractor is considered. The project manager is referred to as she and the contractor is considered as he. This research assumes that the project manager belongs to a large scale organization with the financial resources to be neutral towards the level of the financial risks from the projects whereas, her counterpart contractor belongs to a small scale organization that is more vulnerable to financial risk. Thus, in this context, the project manager is considered to be risk-neutral and the contractor is considered as risk-averse. In order to maintain consistency with the existing literature, the bargaining models are analyzed using utility maximizing supply chain members. The following acronyms are used in this paper.

- z = Wealth value or the pay-off to the member of the supply chain under consideration
- U_{pm}= The project manager's utility
- U_{co}= The contractor's utility
- N (a,b) = Nash product
- a = Fixed part of the cost-sharing contract with a= a_0 for the fixed-price contract and a = a_1 for the cost-plus contract
- b = Variable part of the cost-sharing contract with $b \in [0, 1]$
- X = Cost function as a random variable
- W = Risk exposure of the contractor
- q = The project value
- D = Disagreement point pay-off
- $a_i(z, D)$ = The aspiration point pay-off
- K(z,D) = The Kalai-Smorodinsky solution
- U(z,D) = The utilitarian sum
- η = Constant representing the degree of risk preference of the contractor
- δ = Constant representing the degree of risk preference of the project manager
- ω = Shape parameter of the gamma distributed cost
- ϕ = Scale parameter of the gamma distributed cost or scale for the beta distributed cost
- c & d = Shape parameters of the beta distributed cost
- μ = Mean value of the cost
- σ = Standard deviation of the cost

By definition, the utility function for a risk-neutral member should have a constant marginal return [19,29]. This is satisfied by a linear form of utility function with respect to wealth. Thus, for, the risk-neutral project manager and the risk-neutral contractor, the utility functions are as follows

$$U_{pm}(z) = U_{co}(z) = z \tag{1}$$

On the contrary, a risk-averse contractor's utility function should satisfy the diminishing marginal return [19,29]. This condition is satisfied as long as the utility function is concave in nature [29]. We used the decreasing exponential form of the utility function as it is the most common concave form of utility function [8]. This ensured no change in the risk premium with respect to the absolute risk aversion (We considered a decreasing absolute risk-aversion case in the numerical analysis section as an extension to this original case). Thus, the utility function for the risk-averse contractor was assumed to be

$$U_{co}(z) = 1 - e^{-\eta z} \text{ where } \eta > 0 \text{ for the risk} - \text{averse members} \tag{2}$$

The project manager has a project of value q. She needs to outsource the project to an external contractor and so she offers a cost-sharing contract P. The contractor can either accept or reject the contract. If he rejects the contract, then it would be the subject of further negotiation. The contractor would accept the contract if his utility is at least equal to the disagreement point utility (D). We assume that the disagreement point utility (D) for the members of the supply chain to be zero. When the contractor accepts the contract, he needs to select a resource consumption rate to complete the project. Upon completion, the project manager verifies the cost of completion and makes the payment to the contractor. We also assume that the value of η to be of common knowledge to the members of the supply chain in this case.

For a cost-sharing contract, P takes the form of P = a+ bX, where X is the cost function to the contractor. The cost-sharing contract has two parameters a and b. It is assumed that $a > 0$ and $b \in [0, 1]$. a is the fixed component of the contract. b is the variable component of the contract. When b=0 or b=1, the cost-sharing contract is equivalent to a fixed-price contract and a cost-plus contract respectively. For ease of exposition, the time value of money is ignored from this model. Thus, the expected utility functions can be derived as follows from Eqs. (1) and (2)

$$U_{pm} = E[q - (a + bX)] = q - a - bE(X) \textbf{ For the project manager} \tag{3}$$

$$\begin{aligned} U_{co} &= E[1 - e^{-\eta\{(a+bX)-X\}}] \\ &= 1 - e^{-\eta a}E[e^{\eta(1-b)X}] \textbf{ For the risk} - \textbf{averse contractor} \end{aligned} \tag{4}$$

[Where the X is the cost function (a random variable)].

4. Bargaining models of supply chain coordination with cost-sharing contracts: Nash's bargaining

Using the Nash bargaining approach, the project manager and the contractor would maximize the Nash product N(z, D) as below

$$\begin{aligned} &N(z, D)^* = \max{:}N(a, b) \\ &\text{where } N(a, b) = U_{pm}(a, b)^* U_{co}(a, b) \end{aligned} \tag{5}$$

With $b \in [0, 1]$, the solution of the bargaining process with the risk-neutral project manager and the risk-neutral contractor leads to equal utilities for the project manager and the contractor [33]. More importantly, it can be shown that the solution is the same for all $b \in [0, 1]$ in this case. Due to their simplicity of implementation, in practice, fixed-price contracts are likely to be preferred by members in these cases.

On the contrary, for the case where the risk-neutral project manager and the risk-averse contractor are as defined in Eqs. (3) and (4), Lippman et al. [33] used a normally distributed cost function X. As argued earlier, this is very unlikely in practice. Thus, this research extends the model with different forms of the probability distribution for the cost function X, namely the gamma and the beta distributions.

As defined earlier, the expected value $E(X) = \mu$. Additionally, let W be defined as $W = E[e^{\eta(1-b)X}]$. The value of $E[e^{\eta(1-b)X}]$ can be derived from the moment generating function of the respective distribution of the cost function X. We assume that W represents the expected risk exposure of the contractor. Thus, using these values in Eq. (5) gives

$$N(a, b) = (q - a - b\mu)(1 - e^{-\eta a}W) \tag{6}$$

In order to get the optimal first-best value, differentiating Eq. (6) with respect to a and setting that equal to zero gives the first-order condition as

$$\frac{\partial N}{\partial a} = \eta W(-a - b\mu + q)e^{-\eta a} + We^{-\eta a} - 1 = 0 \tag{7}$$

On the one hand, with b =0, the contract becomes a fixed-price contract. On the other hand, with b=1, the contract becomes a pure cost-plus contract. Thus,

$$\begin{cases} a = a_0 \text{ and } W = W_0 & \text{for fixed-price contracts} \\ a = a_1 \text{ and } W = W_1 & \text{for cost-plus contracts} \end{cases}$$

It can be easily shown when b = 1, then $W_1 = 1$. Both the contracts' (fixed-price and cost-plus) parameters should satisfy the first best condition in Eq. (7). Thus, using the values of a and W from above, the following conditions can be derived as

$$\begin{cases} W_0[\eta(-a_0 + q) + 1]e^{-\eta a_0} - 1 = 0 & \text{For fixed-price} \\ [\eta(-a_1 - \mu + q) + 1]e^{-\eta a_1} - 1 = 0 & \text{For cost-plus}[W_1 = 1] \end{cases} \tag{8}$$

The W value can be calculated as below

$$W = E[e^{tX}] = \begin{cases} \dfrac{1}{(1 - \phi t)^{\omega}} & \text{gamma distributed cost} \\ 1 + \displaystyle\sum_{i=1}^{\infty}\left(\phi^i \prod_{r=0}^{i-1} \frac{c + r}{c + d + r}\right)\frac{t^i}{i!} & \text{beta distributed cost} \end{cases} \tag{9}$$

where $t = \eta(1-b)$, please see Section 3 for the rest of the parameter details

Thus, for a fixed-price contract with $b = 0$, the following can be derived

$$W = W_0 = \begin{cases} \dfrac{1}{(1-\eta\phi)^{\omega}} & \text{gamma distributed cost} \\ 1 + \displaystyle\sum_{i=1}^{\infty}\left(\prod_{r=0}^{i-1}\frac{c+r}{c+d+r}\right)\frac{\phi^i\eta^i}{i!} & \text{beta distributed cost} \end{cases} \tag{10}$$

As stated earlier, $W=1$ for $b=1$ irrespective of any distribution. Now, the mean value μ for the cost functions are as follows

$$\mu = \begin{cases} \omega\phi & \text{gamma distributed cost} \\ \phi\left(\dfrac{c}{c+d}\right) & \text{beta distributed cost} \end{cases} \tag{11}$$

Using these values of W (Including W_0) and μ in Eq. (7), the optimal condition for the contract parameter a can be determined for the selected distributions of the cost function. This is summarized in the following lemmas.

Lemma 1. *The optimal value of contract parameter a_0 of a cost-sharing contract $P= a+bX$ that maximizes the Nash product, satisfies the following conditions if $b = 0$*

$$\begin{cases} \left[\dfrac{1}{(1-\eta\phi)^{\omega}}\right][\eta(-a_0+q)+1]e^{-\eta a_0} - 1 = 0 & \text{gamma} \\ \left[1 + \displaystyle\sum_{i=1}^{\infty}\left(\prod_{r=0}^{i-1}\frac{c+r}{c+d+r}\right)\frac{\phi^i\eta^i}{i!}\right] & \text{beta} \\ \quad [\eta(-a_0+q)+1]e^{-\eta a_0} - 1 = 0 & \end{cases} \tag{12}$$

Lemma 2. *The optimal value of contract parameter a_1 of a cost-sharing contract $P= a+bX$ that maximizes the Nash product, satisfies the following conditions if $b = 1$*

$$\begin{cases} [\eta(-a_1-\omega\phi+q)+1] & \text{gamma distributed cost} \\ \quad e^{-\eta a_1} - 1 = 0 & \\ \left[\eta\left(-a_1 - \dfrac{\phi c}{c+d} + q\right) + 1\right] & \text{beta distributed cost} \\ \quad e^{-\eta a_1} - 1 = 0 & \end{cases} \tag{13}$$

Following Lippman et al. [33]'s suggestion to use the sign test of the derivatives, the Nash product or the individual utility functions of the members of the supply chain can be shown as either increasing or decreasing in b. Accordingly, it can be concluded whether fixed-price, or cost-plus or any other form of cost-sharing contract dominates the solution. Thus, differentiating Eq. (5) with respect to b, we get

$$\frac{dN(a,b)}{db} = U_{co}\left(\frac{dU_{PM}}{db}\right) + U_{PM}\left(\frac{dU_{co}}{db}\right) \tag{14}$$

Differentiating Eq. (3) with respect to b, and using the value of $E(x) = \mu$

$$\frac{dU_{PM}}{db} = -\frac{da}{db} - \mu \tag{15}$$

Rearranging the terms from Eq. (7), we get

$$(q - a - b\mu) = \frac{1-e^{-\eta a}W}{\eta e^{-\eta a}W} = \frac{1-A}{\eta A} \quad [\text{where } A = e^{-\eta a}W] \tag{16}$$

Differentiating Eq. (16) with respect to b, we get the following

$$-\frac{da}{db} - \mu = -\frac{1}{\eta A^2}\frac{dA}{db} \tag{17}$$

Now

$$\frac{dA}{db} = -\eta e^{-\eta a}W\frac{da}{db} + e^{-\eta a}\frac{dW}{db} \tag{18}$$

Using this value of A and $\frac{dA}{db}$ from (18) in Eq. (17)

$$-\frac{da}{db} - \mu = -\frac{1}{\eta A^2}\left[-\eta A\frac{da}{db} + \frac{A}{W}\frac{dW}{db}\right] = -\frac{1}{A}\left[\left(\frac{1}{\eta W}\right)\frac{dW}{db} + \mu\right] \tag{19}$$

As mentioned earlier, $U_{co} = 1 - e^{-\eta a}W$. Based on the assumption made in Eq. (16), $U_{co} = 1 - A$. Thus, differentiating both sides with respect to b

$$\frac{dU_{co}}{db} = -\frac{dA}{db} \tag{20}$$

Thus, the signs of $\frac{dU_{pm}}{db}$ and $\frac{dU_{co}}{db}$ depend on the signs of $\frac{dW}{db}$ and $\frac{dA}{db}$ respectively. The sign tests of these derivatives depend on the nature of the distribution of the cost function X. Using the first-order derivative of W i.e. $\frac{dW}{db}$ for gamma and beta distributed cost in the Eq. (19) we get

$$\left[-\frac{da}{db} - \mu\right] = \begin{cases} \left(\dfrac{1}{1+A}\right)\left[\dfrac{\omega\eta\phi^2(1-b)}{\{1-\eta\phi(1-b)\}}\right] & \text{gamma dist cost} \\ \dfrac{\left[\frac{c\eta(1-b)\phi^2}{(c+d)A}\right]\left[\left(\frac{d}{(c+d)(c+d+1)}\right) + \left(\frac{2d(c+1)\eta(1-b)\phi}{2!\,(c+d)(c+d+1)(c+d+2)}\right) + \cdots\right]}{1 + \eta(1-b)\left[\frac{\phi c}{c+d}\right] + \frac{\eta^2(1-b)^2}{2!}\left[\frac{\phi^2 c(c+1)}{(c+d)(c+d+1)}\right] + \cdots} & \text{beta dist cost} \end{cases} \tag{21}$$

The parameters ω, c, d, and ϕ are assumed to be positive. The value of A as assigned in Eq. (16) is positive. It can be easily shown that $1-\eta(1-b)\phi > 0$ for the gamma distributed cost functions (Please see the appendix for a detailed proof). Hence, from Eq. (21), it can be shown that the right-hand side of the equation becomes positive for $0 \le b < 1$ and zero for $b=1$. Combining this observation with what we found in Eqs. (15) and (17), the situation is summarized in the following lemma (Please see the appendix for a detailed proof)

Lemma 3. *With a cost-sharing contract $P=a+bX$ (where X follows a gamma with shape parameter ω and scale parameter ϕ or beta distribution with shape parameters c and d, and scale parameter ϕ), the Nash product and the utility functions of the risk-neutral project manager & the risk-averse contractor are higher under the cost-plus contract than under the fixed-price contract or any cost-sharing contract ($0 < b < 1$).*

Similarly, the calculations can be extended for cost functions following other continuous distributions such as the exponential, and Weibull. It can be shown that cost-plus contracts are capable of offering a dominating solution to ensure a win-win solution. Hence, from the findings from lemma 3, the following generalization is proposed

Proposition 1. *With a cost-sharing contract $P=a+bX$ (where cost function X follows any non-normal skewed continuous distribution), the Nash product and the utility functions of the risk-neutral project manager & the risk-averse contractor are higher under the cost-plus contract than under the fixed-price contract or any other cost-sharing contract ($0 < b < 1$). The optimal condition for the fixed parameter of the contract satisfies the condition in Eq. (13) in lemma (2) for the gamma and beta distributed costs.*

5. Bargaining models of supply chain coordination with cost-sharing contracts: Kalai-Smorodinsky bargaining

The utility functions for the project manager and the contractor remain the same as described in Eqs. (3) and (4) respectively. According to the Kalai-Smorodinsky rule [27], the optimal solution is

$$K(z, D) = arg \max_{z_i} \left\{ \min_{(i \in pm, co)} \frac{z_i - D_i}{a_i(z, D) - D_i} \right\} \tag{22}$$

Where i denotes either the project manager or the contractor; z_i is the pay off to member i; D_i is the disagreement pay-off; and $a_i(z, D)$ is the aspiration pay-off to member i. $a_i(z, D)$ is defined as $a_i(z, D) = arg\max(z_i)$. Thus, the Kalai-Smorodinsky solution K(z,D) maximizes the individually rational pay-off normalized (U_{pmn} for the project manager and U_{con} for the contractor in this case) with respect to the aspiration point pay off.

As before, the disagreement pay-off (D_i) for both the members is assumed as zero. The aspiration point for the project manager and the contractor are respectively as follow

For the project manager

$$a_{pm}(z, D) = q - E(X) = q - \mu \tag{23}$$

For the contractor

$$a_{co}(z, D) = \begin{cases} q - \mu & \text{Risk} - \text{neutral contractor} \\ 1 - e^{-\eta q} E\{e^{\eta X}\} = 1 - e^{-\eta q} V & \text{Risk} - \text{averse contractor} \end{cases} \tag{24}$$

[where $\mu = E(X)$ and $V = E\{e^{\eta X}\}$]

Similar to the case of Nash's bargaining with risk-neutral members, for Kalai-Smorodinsky bargaining the maximum utility is equally split amongst the members. Like Nash's bargaining, this is the same for fixed-price and cost-plus contracts. Thus, due to its simplicity, the fixed-price contract is likely to be preferred over the cost-plus contract in practice in this situation.

On the contrary, for the case with the risk-neutral project manager and the risk-averse contractor, using these aspiration point values, the normalized individual rationalities of the members of the supply chain are calculated as follows

$$U_{pmn} = \frac{q - a - b\mu}{q - \mu} \quad \textbf{For the project manager} \tag{25}$$

$$U_{con} = \frac{1 - e^{-\eta a} W}{1 - e^{-\eta q} V} \quad \textbf{For the contractor} \ \text{[W follows equation (9)]} \tag{26}$$

In order to satisfy the condition for optimal K, the minimum values of the two fractions on the right hand side of the Eqs. (25) and (26) should be maximized. When the minimum of these two fractions are maximized, they become equal in value. Thus, at the optimal solution

$$\frac{q - a - b\mu}{q - \mu} = \frac{1 - e^{-\eta a} W}{1 - e^{-\eta q} V} \tag{27}$$

As defined earlier, $W = E(e^{\eta(1-b)X})$. Thus, (for b = 0), $V = W_0$ as V is assumed as $E(e^{\eta X})$. Moreover, it is defined in Section 4 that $b = 1, a = a_1$ for a cost-plus contract, and $W = 1$. On the contrary, $b = 0, a = a_0$ for a fixed-price contract; and $W = W_0$. Thus using these values in the optimal condition for the Kalai-Smorodinsky Solution in Eq. (27), the optimal conditions are summarized in the lemmas

Lemma 4. *The optimal value of contract parameter a_0 of a cost-sharing contract satisfies the following when the Kalai-Smorodinsky bargaining approach is followed with b = 0*

$$a_0 = \begin{cases} \dfrac{q - a_1 - \omega\phi}{q - \omega\phi} - \dfrac{1 - e^{-\eta a_0} \frac{1}{(1-\eta\phi)^{\omega}}}{1 - e^{-\eta q} \frac{1}{(1-\eta\phi)^{\omega}}} = 0 & \text{gamma dist cost} \\ \dfrac{q - a_0 - \frac{\phi c}{c+d}}{q - \frac{\phi c}{c+d}} & \text{beta dist cost} \\ \quad - \dfrac{1 - e^{-\eta a_0}\left[1 + \sum_{i=1}^{\infty}\left(\prod_{r=0}^{i-1} \frac{c+r}{c+d+r}\right)\frac{\phi^i \eta^i}{i!}\right]}{1 - e^{-\eta q}\left[1 + \sum_{i=1}^{\infty}\left(\prod_{r=0}^{i-1} \frac{c+r}{c+d+r}\right)\frac{\phi^i \eta^i}{i!}\right]} = 0 & \end{cases} \tag{28}$$

Lemma 5. *The optimal value of contract parameter a_1 of a cost-sharing contract satisfies the following when the Kalai-Smorodinsky bargaining approach is followed with b = 1*

$$a_1 = \begin{cases} \dfrac{q - a_1 - \omega\phi}{q - \omega\phi} - \dfrac{1 - e^{-\eta a_1}}{1 - e^{-\eta q} \frac{1}{(1-\eta\phi)^{\omega}}} = 0 & \text{gamma dist cost} \\ \dfrac{q - a_1 - \frac{\phi c}{c+d}}{q - \frac{\phi c}{c+d}} & \text{beta dist cost} \\ \quad - \dfrac{1 - e^{-\eta a_1}}{1 - e^{-\eta q}\left[1 + \sum_{i=1}^{\infty}\left(\prod_{r=0}^{i-1} \frac{c+r}{c+d+r}\right)\frac{\phi^i \eta^i}{i!}\right]} = 0 & \end{cases} \tag{29}$$

In order to identify if the solution with the fixed-price or the cost-plus contract or any other cost-sharing contract ($0 < b < 1$) dominates, the sign tests for the first-order derivatives of the utilities of the project manager $\left(\frac{dU_{pm}}{db}\right)$ and the contractor $\left(\frac{dU_{co}}{db}\right)$ are required. To determine the sign on the right hand side of the above equation, both sides of the Eq. (27) are differentiated with respect to b

$$\left(\frac{-\frac{da}{db} - \mu}{q - \mu}\right) = \left[\eta e^{-\eta a} W \frac{da}{db} - e^{-\eta a} \frac{dW}{db}\right]\left(\frac{1}{1 - e^{-\eta q} V}\right) \tag{30}$$

Rearranging the above gives

$$\left(-\frac{da}{db} - \mu\right)(1 + \eta e^{-\eta a} W B) = \left[-e^{-\eta a}\frac{dW}{db} - \eta e^{-\eta a} W \mu\right] B \tag{31}$$

[where $B = \frac{q - \mu}{1 - e^{-\eta q} V}$]

Now the term B is positive as the maximum possible utilities of the members have to be positive for the participation of the members. The values of W and $\frac{dW}{db}$ would change depending on the nature of the distribution of X. Using the values of W and $\frac{dW}{db}$ for the gamma and beta distributed costs, we get the following

$$\left(-\frac{da}{db} - \mu\right) = \begin{cases} \dfrac{B\eta e^{-\eta a} W \left[\frac{\eta\mu(1-b)\phi}{\{1 - \eta(1-b)\phi\}}\right]}{(1 + B\eta e^{-\eta a} W)} & \text{gamma} \\ \dfrac{B e^{-\eta a} \eta\mu}{(1 + B\eta e^{-\eta a} W)}\left[\left\{\dfrac{d\eta(1-b)\phi}{(c+d)(c+d+1)}\right\} + \right. & \text{beta} \\ \left. \left\{\dfrac{2d(c+1)\eta^2(1-b)^2\phi^2}{2!(c+d)(c+d+1)(c+d+2)}\right\} + \cdots\right] & \end{cases} \tag{32}$$

The value of B has to be positive as it is assigned as the ratio of the maximum value of the utilities of the members of the supply chain (The maximum values of the utilities have to be positive for participation purposes). Following the same assumptions and arguments from Section 4, from Eq. (32), it can easily be shown that the right-hand side of the equation is positive for $0 \le b < 1$ and 0 for b = 1. Now differentiating both sides of Eq. (30), and replacing the values of $\frac{dU_{co}}{db}$, it can be shown that $\frac{dU_{co}}{db} \ge 0$ for $b \in [0, 1]$. Hence, combining these observations with the findings from Eq. (15), the situation is summarized in the following lemma (Please see the appendix for a detailed proof)

Lemma 6. *With a cost-sharing contract $P = a + bX$ (where X follows a gamma with shape parameter ω and scale parameter ϕ or beta distribution with shape parameters c and d, and scale parameter ϕ), the Kalai-Smorodinsky value K, and the utility functions of the risk-neutral project manager and the risk-averse contractor are higher under the cost-plus contract ($b = 1$) than under the fixed-price contract ($b = 0$) or any other cost-sharing contracts ($0 < b < 1$).*

Similarly, the calculations can be extended for cost functions following other continuous distributions such as the exponential and Weibull. It can be shown that cost-plus contracts are capable of offering a dominating solution to ensure a win-win solution. Hence, from the findings from lemma 6, the following generalization is proposed

Proposition 2. *Using the Kalai-Smorodinsky bargaining with a cost-sharing contract $P = a + bX$ (where X can follow any non-normal continuous distribution, and a & b are contract parameters), the solutions for the Kalai-Smorodinsky value K, and the utilities of the risk-neutral project manager & the risk-averse contractor are the dominant solution for a cost-plus contract ($b = 1$). This dominates the solutions from any cost-sharing contract ($0 < b < 1$) and fixed-price contract ($b = 0$). The optimal condition for the fixed parameter of the contract satisfies the condition in Eq. (29) in lemma (5) for the gamma and beta distributed costs.*

6. Bargaining models of supply chain coordination with cost-sharing contracts: Utilitarian approach to bargaining

According to the Utilitarian rule, the sum of the utilities during the bargaining negotiation is maximized. Thus,

$$U(z, D) = arg \max_{u \in z} \sum U_i \qquad (\text{ for all the members i.e. i=pm, co}) \tag{33}$$

In a similar way to the previous calculations of supply chains with both risk-neutral members, the solution is indifferent to fixed-price or cost-plus contracts for the utilitarian approach. Due to the simplicity, members of the supply chain are likely to be inclined to use fixed-price contracts in practice. For the case with a risk-neutral project manager and a risk-averse contractor, substituting the utility functions from Eqs. (3) and (4) into Eq. (33) gives

$$U(z, D) = arg \max_{u \in z} [(q - a - b\mu) + (1 - e^{-\eta a} W)] \tag{34}$$

In order to get the optimal solutions for contract parameter a, Eq. (34) is differentiated with respect to a and then it is set equal to zero as below.

$$\frac{dU(z, D)}{da} = -1 + \eta e^{-\eta a} W = 0$$

Rearranging the terms of the above equation, the first-order condition for a is as follows

$$a = \frac{1}{\eta} \log_e(\eta) + \frac{1}{\eta} \log_e(W) \tag{35}$$

In order to find the optimal conditions for b ($0 \leq b \leq 1$), Eq. (34) is differentiated with respect to b and rearranging the terms gives

$$\frac{dU(z, D)}{db} = \left(\frac{da}{db}\right)(\eta e^{-\eta a} W - 1) - \mu - e^{-\eta a}\left(\frac{dW}{db}\right) \tag{36}$$

Differentiating Eq. (35) gives

$$\frac{da}{db} = \frac{1}{\eta W}\left(\frac{dW}{db}\right) \tag{37}$$

Thus, using this value of $\frac{da}{db}$ Eq. (36) becomes

$$\frac{dU(z, D)}{db} = -\frac{1}{\eta W}\left(\frac{dW}{db}\right) - \mu \tag{38}$$

The values of W and $\frac{dW}{db}$ would vary depending on the nature of the distribution (see Eq. (9)). Using the values of W and $\frac{dW}{db}$ for the gamma and beta distributed costs in the equation above (38), we get the following

$$\frac{dU(z, D)}{db} = \begin{cases} \dfrac{\mu\eta(1-b)\phi}{\{1 - \eta(1-b)\phi\}} & \text{gamma dist} \\ \dfrac{\mu}{W}\left[\left\{\dfrac{d\eta(1-b)\phi}{(c+d)(c+d+1)}\right\} + \right. & \text{beta dist} \\ \quad \left. \left\{\dfrac{2d(c+1)\eta^2(1-b)^2\phi^2}{2!(c+d)(c+d+1)(c+d+2)}\right\} + \cdots \right] & \end{cases} \tag{39}$$

It can be shown that the right-hand side of Eq. (39) is positive for $0 \leq b < 1$ and 0 for b=1. Differentiating U_{pm} with respect to b and using the values of μ for the gamma and beta distributed costs, it can be shown that $\frac{dU_{pm}}{db} = \frac{dU(z,D)}{db}$ and $\frac{dU_{co}}{db} = 0$. Using this observation, the situation is summarized in the following lemma (Please see the appendix for detailed proof)

Lemma 7. *With a cost-sharing contract $P = a + bX$ (where X follows a gamma with shape parameter ω and scale parameter ϕ or beta distribution with shape parameters c and d, and scale parameter ϕ), the Utilitarian sum U, and the utility functions of the risk-neutral project manager are higher under the cost-plus contract ($b = 1$) than under the fixed-price contract ($b = 0$) or any other cost-sharing contracts ($0 < b < 1$). However, the risk-averse contractor's utility does not change for any values of b for $b \in [0, 1]$.*

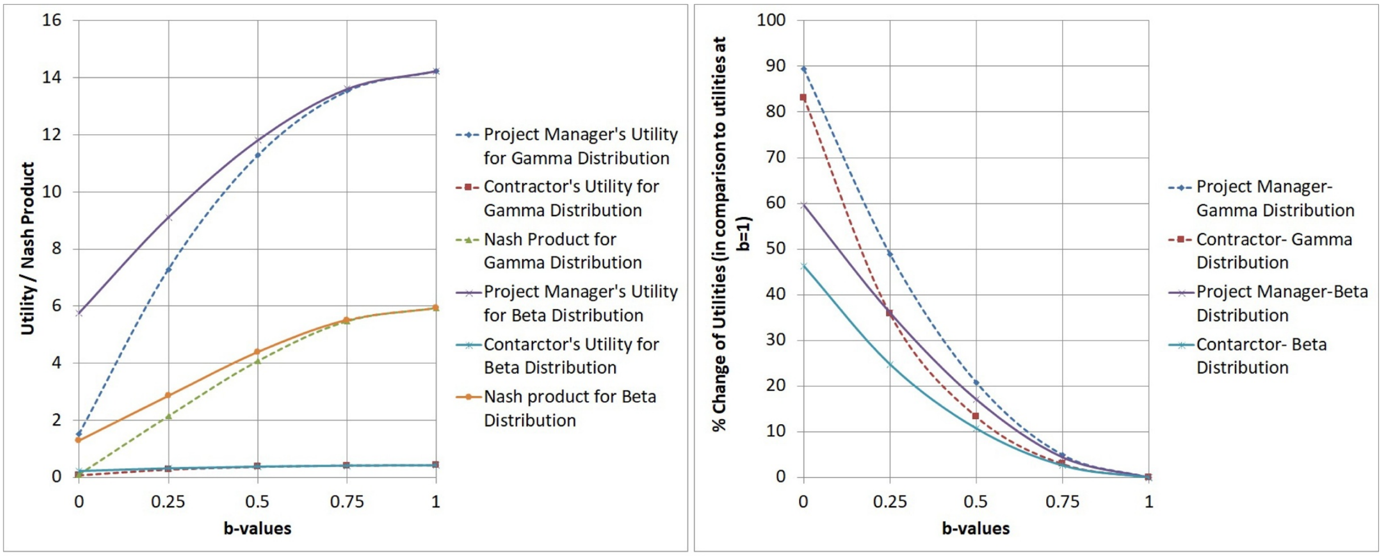
Similarly, the calculations can be extended to cost functions following other continuous distributions such as the exponential and Weibull. It can be shown that cost-plus contracts are capable of offering a dominating solution to ensure a win-win solution for the project manager and the contractor's utility remains unchanged for $b \in [0, 1]$. Hence, from the findings from lemma 7, the following generalization is proposed

Proposition 3. *Using the utilitarian bargaining approach for any non-normal continuous distribution with a cost-sharing contract $P = a + bX$, the solutions derived from a cost-plus contract ($b = 1$) dominates over the solutions derived from any other cost-sharing contract ($0 < b < 1$) or a fixed-price contract ($b = 0$) for the utilitarian sum U(z,D) and the utility value of the risk-neutral project manager. However, the utility value for the risk-averse contractor remains the same for $b \in [0, 1]$. The optimal value of a satisfies the condition in Eq. (35). The W value in Eq. (35) satisfies the conditions in Eq. (9).*

Following from the Proposition 3, the contractor is indifferent to the value selected for b when $b \in [0, 1]$ if his objective is to maximize his utility from the contract offered by the project manager. Hence, if the contractor has a secondary objective of maximizing his profit or if the contractor is less risk-averse than the project manager perceives, then there is a chance that the contractor may prefer the b which is not 1 for $b \in [0, 1]$. Thus, depending on the bargaining power of the members of this supply chain, the negotiation may result in an outcome which is not win-win for both.

7. Numerical analysis and results

This section illustrates the bargaining models using numerical examples. We used an Australian electricity company's dataset from their construction projects. The dataset was retrieved from the paper by Jackson [25]. The authors derived the standard deviation of the completion costs as a percentage of the mean. Hence, for ease of exposition, we assume the mean completion cost to be $\mu = \$100k$. The average value of the standard deviation has been computed as 26%. According



(a) Individual Utilities/Nash Product for different b values (b) Efficiency of the Win-Win Solution

Fig. 1. Individual Utilities/Nash Product for different b values.

to Potts and Ankrah [40], construction projects have low margins. Thus, we assume the cost to be 80% of the original project value and we calculate this to be q = $125k.

The models are derived using gamma and beta distributed costs. The parameter η is assumed as 0.05 in the beginning. The other distribution specific values are assumed as below. With the information (mean and standard deviation) as above, for a gamma distributed cost, the shape parameter becomes ω = 14.80 and the scale parameter becomes ϕ = 6.76. Firstly, the value of W is calculated for the gamma distribution using the numeric values. For construction projects, beta distributed costs are usually coupled with Project Evaluation Review Techniques (PERT). Davis [11] proved that the sum of the shape parameters should be between 4 and 8 i.e. $4 \leq c + d \leq 8$. With the mean and standard deviation values mentioned earlier, using Excel's Solver, we calculated the shape parameters as c = 4.97 and d = 3.03. The scale is calculated as ϕ = 160.84.

7.1. Nash bargaining

Using the conditions from equation (8), the optimal value of a is calculated for b = 0, 0.25, 0.5, 0.75 and 1. Using these values, the values of U_{pm}, U_{co} and Nash product N are calculated. The results are presented in Fig. 1(a).

From Fig. 1(a), it was found that the value of the U_{pm} was 1.51 (5.76) units at b = 0 for the gamma (beta) distributed cost. Then it increased monotonically and became 14.24 units at b = 1 for both the cost distributions. Similar is the case for U_{co} with 0.07 (0.22) units at b = 0 for gamma (beta) distributed cost, then increased monotonically, and finally became 0.42 units at b = 1 for both the cost distributions. As a result of the changes in U_{pm} and U_{co}, the Nash product increased monotonically for $b \in [0, 1]$. For b = 0, the Nash product was 0.11 (1.28) units for the gamma (beta) distributed cost and at $b = 1$, it was 5.93 units for both the cost distributions. Thus, the results of a cost-plus contract (at $b = 1$) dominates the solutions from a fixed-price contract (at $b = 0$) or any other cost-sharing contract with $0 < b < 1$. This supports the original findings from the models. Comparing the results of the two distributions, we found a similar movement of the functions U_{pm}, U_{co}, and N. However, the difference of utilities for $b = 0$ (fixed price contract) and for $b = 1$ (cost-plus contract) is higher in the case of gamma distributed cost. This could be explained by the nature of the distributions. The upper limit of the gamma distributed cost is unbounded whereas it is bounded for the beta distributed cost. Having an unbounded upper limit would leave the possibility of a higher incurred cost than the bounded upper limit. This was perceived as more risky leading to a greater amount of perceived benefit of using a cost-plus contract over a fixed-price contract. Hence, for the same mean and standard deviation, the impact of using the cost-plus contract was more significant over the fixed cost in this unbounded case (gamma distributed cost function).

We further presented the efficiency of the win-win solution of selecting $b = 1$ over the other cost-sharing contracts with $0 \leq b < 1$ in the form of comparison in Fig. 1(b). If the project manager deems the cost-plus contract i.e. $b = 1$ to be inappropriate due to the lack of control over the contractor's expenditure and any opportunistic behavior, then she would lose up to approximately 90% and 60% of her utility (at $b = 0$) for the gamma and beta distributed cost functions respectively. For other cases ($0 < b < 1$) where the project manager shares a part of the cost of the contractor, the loss of utility may not be as significant as 90%, but may still be significant in most cases. However, for values of $b \geq 0.75$, this utility loss is less than 10% for both the contractor and the project manager for both the distributions. In practice, any project manager would try to avoid these huge utility losses. However, she would be required to make the trade-off between the trust she has on the contractor not being opportunistic to exaggerate the cost, and the loss she would incur for selecting a cost-plus contract ($b = 1$). If her fear of being exploited becomes a dominant decision making factor, that would lead her to become risk-averse and in that case, her decisions for the selection of the b value will change.

7.2. Kalai-Smorodinsky bargaining

Using the parameter values assumed earlier, the values of U_{pm}, U_{co}, and the Kalai-Smorodinsky Function K are determined for $b = 0$, 0.25, 0.5. 0.75 and 1. The results are presented in the distributed cost Fig. 2(a).

Similar to the observation in the case of Nash's bargaining, the values of U_{pm}, U_{co}, and K are found to be increasing in the value of b when $b \in [0, 1]$. From Fig. 2(a), it was found that the value of the U_{pm} was 2.67 (7.96) units at $b = 0$ for the gamma (beta) distributed cost. Then it increased monotonically and became 22.37 (17.89) units at $b = 1$ for both the gamma (beta) distributed cost function. Similar is the case for U_{co} which was 0.01 (0.13) units at $b = 0$ for gamma (beta) distributed cost, then increased monotonically, and finally became 0.12 (0.30) units at $b = 1$ for the gamma (beta) distributed cost function. As a result, the K values increased monotonically from 0.11 (0.32) for the gamma (beta) distributed cost at $b=0$ to 0.89 (0.72) for gamma (beta) distributed cost at $b=1$. Thus, once again the results of a cost-plus contract dominate the solutions from a fixed-price contract and from any other cost-sharing contract with $0 < b < 1$. This once again supports the original findings from the model.

Again similar distribution specific impacts on the functions U_{pm}, U_{co}, and K were observed as we found in the case of Nash bargaining. Again, the unbounded gamma distributed cost has a non-zero chance of having higher completion cost at a specific value which is not the case with the bounded beta distributed cost. This was again perceived to be riskier leading to a greater amount of perceived benefit of using a cost-plus contract over a fixed-price contract for the gamma distributed cost over the beta.

We further presented the efficiency of the win-win solution of selecting $b=1$ over the other cost-sharing contracts with $0 \le b < 1$ in the form of comparison in Fig. 2(b). The project manager and the contractor both would lose up to approximately 90% and 55% of her utility (at $b=0$) for the gamma and beta distributed cost functions respectively. For other cases ($0 < b < 1$) where the project manager shares a part of the cost of the contractor, the loss of utility may not be as significant, but is still considerable in many cases. In practice, any project manager would try to avoid a huge utility loss. Hence, again, the project manager would be required to manage the trade-off we mentioned in the case of Nash bargaining.

7.3. Utilitarian bargaining

Using the optimal value of a from Eq. (35) in the equation of the utility function of the contractor and rearranging the terms

$$U_{co} = 1 - \frac{1}{\eta} \tag{40}$$

As mentioned earlier, $U_{co} > 0$, otherwise the contractor will not participate in the bargaining. Thus, for the utilitarian bargaining approach $\eta > 1$. Thus, unlike the case of Nash's bargaining and Kalai-Smorodinsky bargaining, the risk aversion parameter η cannot take lower values. To put it in other words, the utilitarian approach is applicable for more risk-averse members than the cases we explored for Nash and Kalai-Smorodinsky bargaining cases.

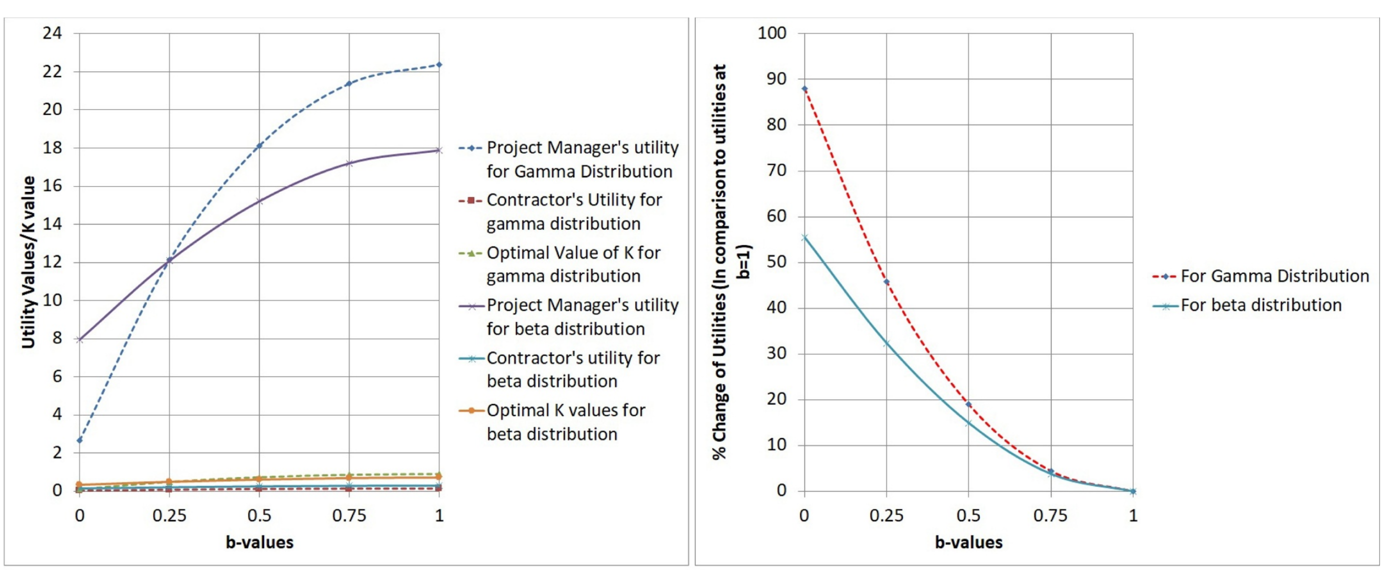
μ and q are assumed as \$100k and \$125k respectively as before. However, η is assumed as 1.3. Moreover, we assumed the σ as 2.69% instead of 26%. This was because the derived models were generating undefined utilities for standard deviations with a higher θ for $0 \le b < 1$. In practice, this can be explained as the non-applicability of the models in the cases of higher risk and lower risk-aversion. Thus, this would become another limitation of using the utilitarian approach. Again, the analysis was conducted for gamma, and beta distributed cost. For the gamma distributed cost, the parameters are calculated as $\omega = 14.80$ and $\phi = 0.7$. The parameters for the beta distributed costs are calculated as $c = 4.97$, $d = 3.02$ and $\phi = 16.66$. Using these, the values of U_{pm}, U_{co}, and the Utilitarian Sum are determined for $b=0$, 0.25, 0.5. 0.75 and 1. The results are presented in distributed cost Fig. 3(a). We found some similarity in results for the utilitarian sum and the utility of the project manager in comparison to what we found in the cases of Nash bargaining and the Kalai-Smorodinsky bargaining. Both those functions (U and U_{pm}) were found to be monotonically increasing with the increase in b for $b \in [0, 1]$. However, unlike the previous cases, the contractor's utility remained the same ($U_{co} = 0.23$) throughout for $b \in [0, 1]$.

Again some distribution specific impacts on the functions U_{pm} and U were observed. The impact of using cost-plus contracts on the project manager's utility and on the utilitarian sum was more significant over using the fixed-price contracts for the gamma distributed cost functions. Again this was due to the same distribution specific characteristics explained in the case of Nash bargaining. However, the impact of using the cost-plus contracts over fixed price was found to have no effect for both the distributions. In fact, it even remained unchanged for gamma and beta distributed costs as long as η remained the same. This can be explained from the Eq. (40). The contractor's utility only depended on his risk perception parameter η.

We further presented the efficiency of the win-win solution of selecting $b=1$ over the other cost-sharing contracts with $0 \le b < 1$ in the form of comparison in Fig. 3(b). The project manager would lose up to approximately 15% and 2.5% of her utility (at $b=0$) for the gamma and beta distributed cost functions respectively. In practice, any project manager would try to avoid any huge utility loss. Hence, again, the project manager would be required to manage the trade-off we mentioned in the case of Nash bargaining.

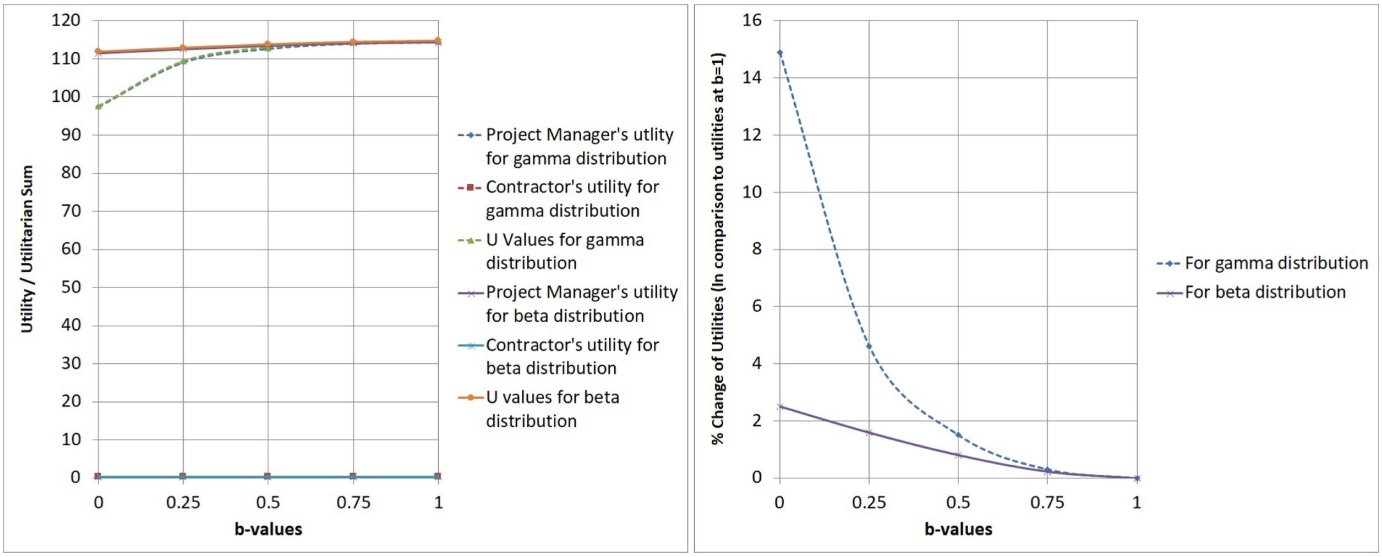
7.4. Comparison between the three bargaining approaches

We found similarities and dissimilarities between the results of the three bargaining approaches we used for modeling. In both the cases of



(a) Individual Utilities/K for different b values (b) Efficiency of the Win-Win Solution

Fig. 2. Individual Utilities/Kalai-Smorodinsky value K different b values.



(a) Individual Utilities/U for different b values (b) Individual Utilities/U for different b values

Fig. 3. Individual Utilities/Utilitarian sum value U for different b values.

Nash and Kalai-Smorodinsky bargaining approaches, the solutions with a cost-plus contract (b = 1) were found to be dominating the other solutions for both the project manager and the contractor. In fact, the project manager (the contractor) was found to be losing up to 90% of her utility for not selecting the cost-plus contract in certain cases of the gamma distributed cost. However, the percentage loss of utilities for not selecting $b = 1$ was different for the project manager and the contractor for the Nash bargaining approach, but the same for the Kalai-Smorodinsky bargaining approach. This was due to the normalization applied as part of the Kalai-Smorosinsky approach. Nevertheless, the trade-off for the project manager (between not trusting the contractor to avoid the opportunistic behavior of cost exaggeration and losing a significant amount of her utility) would generate significantly different results upon selection of the extremes of the trade-off in both of these bargaining cases.

On the contrary, the utilitarian bargaining approach, showed somewhat different results for the cases explored. The contractor did not have any change in his utility for $b \in [0, 1]$ and hence, no loss of utility for not selecting $b = 1$. The percentage loss of utility for the project manager not selecting $b = 1$ was not as significant as we found in the other two bargaining approaches (Please see Fig 3(b)). Thus, the trade-off for the project manager (between not trusting the contractor to avoid the opportunistic behavior of cost exaggeration and losing a significant amount of her utility) would not generate such significantly different results as found in the other two bargaining cases. Thus, the project manager may have some indecision over selecting the correct value of $b \in [0, 1]$. Furthermore, the project manager's (the contractor's) profit was found to be increasing (decreasing) with any increase in b for $b \in [0, 1]$. Due to the indifference in her utility values, if the contractor has a secondary objective of profit maximization apart from his primary one of reducing the risk, then he might prefer the fixed price contract. Summarizing the discussion of this paragraph, the use of the utilitarian bargaining approach may lead to some inconclusive results. This suggests that the win-win solution may be difficult to be reached, and hence, this approach may be difficult to be managed in practice.

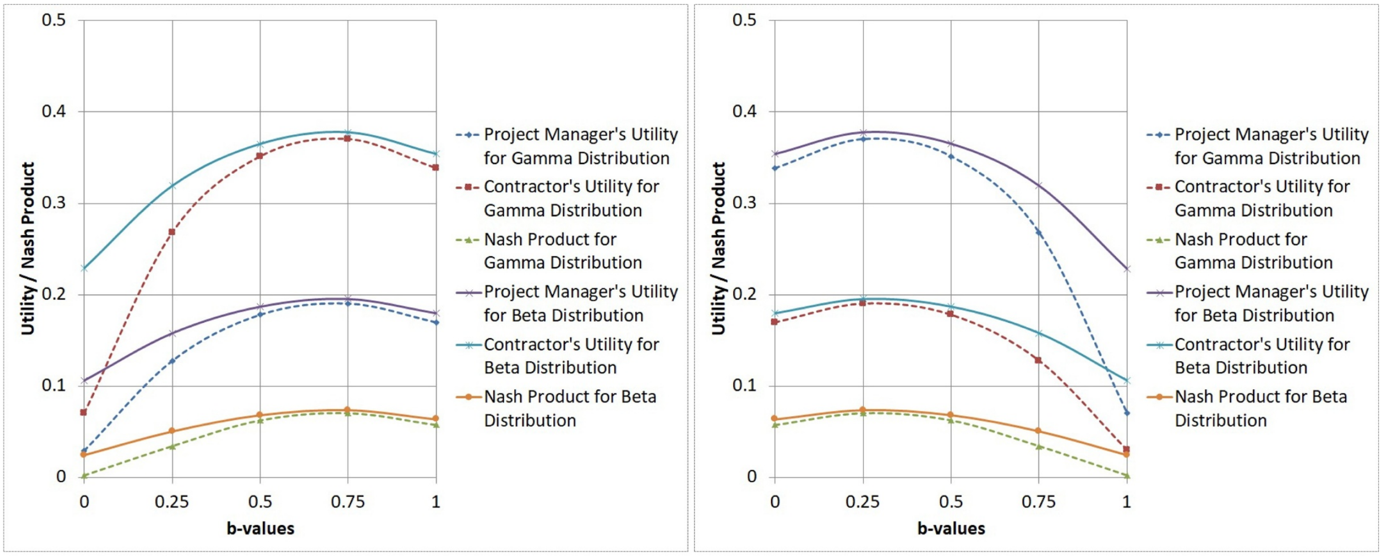
7.5. Further sensitivity analysis

In Sections 7.1, 7.2, and 7.3, we presented the cases with only the contractor as risk-averse whereas the project manager as risk-neutral. In this section, we consider the case when the project manager is risk-averse too. We denote her degree of risk preference as δ (as stated earlier). Her utility function becomes $U_{pm} = 1 - e^{-\delta(q-a)}M$ (assuming the risk-averse project manager will also have an exponential utility function like the risk-averse contractor. $M = E[e^{\delta bX}]$ is the expected risk exposure of the project manager).

For the case of Nash bargaining, the project manager and the contractor would try to optimize the Nash product $N = (1 - e^{-\delta(q-a)}M)(1 - e^{-\eta a}W)$. The optimal a value satisfies the condition $-\{\delta M e^{\eta a} - \eta W e^{\delta(q-a)} + (\eta - \delta)MW\}e^{-\eta a - \delta(q-a)}$. We further analyzed the optimal b value numerically and the results are presented in Figs. 4(a) and (b).

From these figures, we found that the optimal value of b is neither 0 nor 1, but somewhere between 0 and 1 when both the project manager and the contractor are risk-averse. We analysed two cases: when the project manager was less risk-averse than the contractor ($\delta = 0.02$ and $\eta = 0.05$), and vice-versa ($\delta = 0.05$ and $\eta = 0.02$). When the project manager is less risk-averse than the contractor, the optimal solution is around $b = 0.75$ to 0.8 i.e. she is ready to share 75% to 80% of the contractor's cost as an optimal solution. This reduction in b value in comparison to the base case of the risk-neutral project manager was due to the project manager becoming risk-averse. We further investigated the case with the project manager becoming more risk-averse (with $\delta = 0.05$) and the contractor becoming less risk-averse(with $\eta = 0.02$). We found that the optimal $b = 0.25$ to 0.3 the project manager preferred to share approximately 25% to 30% of the contractor's cost. It can be easily shown that when the project manager becomes risk-averse and the contractor risk-neutral, the optimal solution becomes a fixed-price contract. This can be explained from practical considerations - as the project manager became more risk-averse, she preferred to avoid any risk of an excessive cost from the contractor being passed over to her (please see the issues around trust in Section 7.1). This is why she preferred contracts closer to a fixed price when she started to become more risk-averse. It can be easily shown when both the project manager and the contractor are equally or almost equally risk-averse, the optimal solution of b is around 0.5 i.e. they negotiate to share approximately 50% of the cost incurred by the contractor.

Similar results were also found in the Kalai-Smorodinsky case. The optimal a value satisfied the condition $\frac{1 - e^{-\delta(q-a)}M}{1 - e^{-\delta q_L}} = \frac{1 - e^{-\eta a}W}{1 - e^{-\eta q_V}}$ (M is defined earlier in this section; L is assumed as the $L = E[e^{\delta X}]$) which are



(a) For $\delta = 0.02$ and $\eta = 0.05$ (b) For $\delta = 0.05$ and $\eta = 0.02$

Fig. 4. Individual Utilities/Nash Product for different b values: both risk averse members.

presented in Figs. 5 (a) and (b).

For the Utilitarian bargaining approach, for various values of δ and η, the proposed model failed to offer solutions for some of the values of $b \in [0, 1]$. However, for most of the cases, we found that $b = 0.5$ gave the optimal value of both the project manager's and the contractor's utilities.

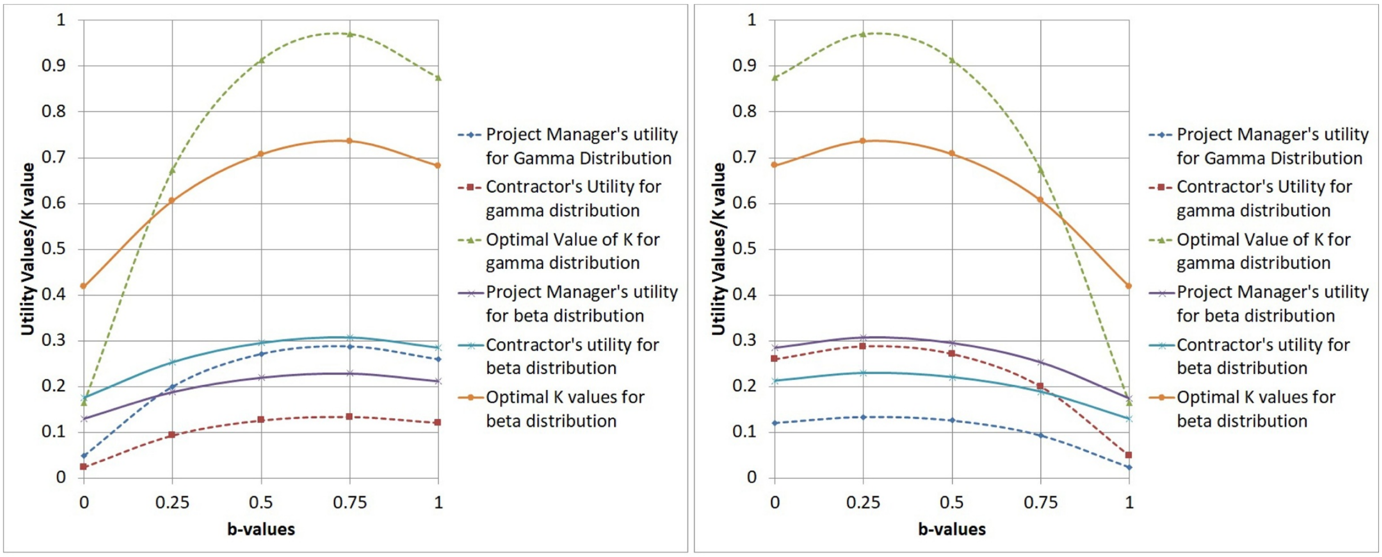
We also conducted a sensitivity analysis of when the utility function of the risk-averse contractor is not an exponential function keeping the project manager risk-neutral as before. We considered $U_{co}(z) = \frac{z^{1-\eta}}{1-\eta}$ (For $z > 0$ and $\eta \neq 1$. For $\eta = 1$, the function would become $U_{co} = \ln z$); so the utility of the contractor becomes $U_{co} = \frac{\{a-(1-b)X\}^{1-\eta}}{1-\eta}$. This type of risk-aversion function is used for decreasing absolute risk aversion (DARA) cases. This ensures that the risk-averse contractor will require less risk-premium with the increase in wealth value z. We did not consider the increasing absolute risk-aversion case as the risk premium will continue to increase with the increase in the wealth, and so the original results are less likely to change. We kept the risk-neutral project manager's utility as considered before. The results are generated with the assumption that $a > (1 - b)X$, i.e. the project manager needs to offer a minimum contract value that is greater than the incurred cost by the project manager. We also used the same set of parameter values as assumed in sections 7.1 and 7.2. The results for the Nash bargaining and Kalai-Smorodinsky bargaining cases are presented in Fig. 6(a) and (b) respectively. We found that the cost-plus contracts are still dominating the solutions i.e. the best possible options for both the project manager and the contractor. However, the change in utilities with respect to the fixed price or any other cost-sharing contracts, for using the cost-plus contract was found to be very low. This was due to the fact that the contractor's risk aversion function was a decreasing absolute risk aversion and he was prepared to invest more money or expecting less risk premium for his participation. As a result, the expected benefits were less than the base case. The results for a Utilitarian bargaining case (presented in Fig. 6(c)) is somewhat different than we found in the case of the Nash and Kalai-Smorodinsky bargaining case. Unlike the original case presented in Section 7.3, the proposed models failed to generate any meaningful result for $\eta > 1$. Hence, we considered $\eta = 0.05$ keeping the rest of the parameters the same as assumed in Section 7.3. The cost-plus contract was found to be very marginally more beneficial (in comparison to the fixed price) for the project manager, but it was marginally less beneficial (in comparison to the fixed price) for the contractor than in comparison. As a result, our model failed to get a unique win-win solution in this case. In real projects, if the members use this type of bargaining, then the outcome would be subject to their individual bargaining power.

8. Discussions and concluding remarks

This paper has analyzed the bargaining solution for projects where the contractual payment from the project manager to the contractor is a linear function of the contractor's cost. The important categories of fixed price and cost-plus contracts are included in this type of contract. It was assumed that the project manager was risk-neutral and the contractor was risk-averse - this corresponds to the common situation of a large project manager and a significantly smaller contractor. The key contributions to the knowledge include

- The derivation of mathematical models for project supply chain bargaining situations that can establish an optimal win-win solution with differential risk perceptions of the supply chain members when the cost functions follow more realistic distributions (gamma and beta).
- The Nash and Kalai-Smorodinsky bargaining approaches have a similar impact on the win-win results, while the Utilitarian approach gives different results.
- An investigation of the difficulties a utilitarian bargaining approach has on generating the win-win outcome as it leads to inconclusive results in certain cases.

There are certain managerial implications that can be drawn from the findings of this paper. The first one is the analysis of more realistic distributions for the cost functions. Gamma and beta distributions were used to model the contractor's cost as they have been found to be a better model of project costs than the previously used normal distribution [17,43]. A comparison between the two types of cost distributions revealed some further managerial implications. Due to the right tail being unbounded, the gamma distributed cost functions had a non-zero probability of having an unknown maximum value of completion cost. This led to a greater cost uncertainty. This did not apply to the beta distributed cost due to its bounded nature. Thus, for these cases, the project manager was required to offer a greater amount of surplus to entice the contractor to participate. Especially in these types of cases of uncertainty (with unknown maximum cost), the benefit of



(a) For $\delta = 0.02$ and $\eta = 0.05$

(b) For $\delta = 0.05$ and $\eta = 0.02$

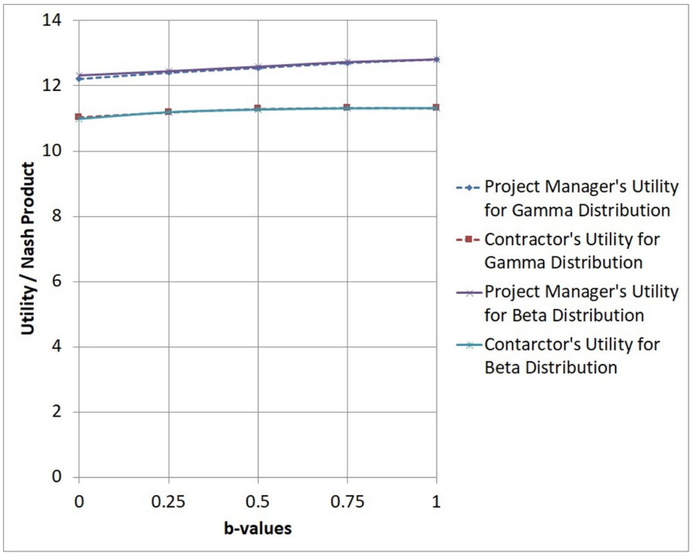
Fig. 5. Individual Utilities/K for different b values: both risk-averse members.

using a cost-plus contract over the fixed price contract was significantly higher than with the cases of having a known upper bound of the cost. Similar results can be derived for other continuous distributions following the same set of procedures highlighted in this paper for the cases of the gamma and beta distributed cost. The results would vary a little depending on the type of the distribution (if bounded on the right tail or not).

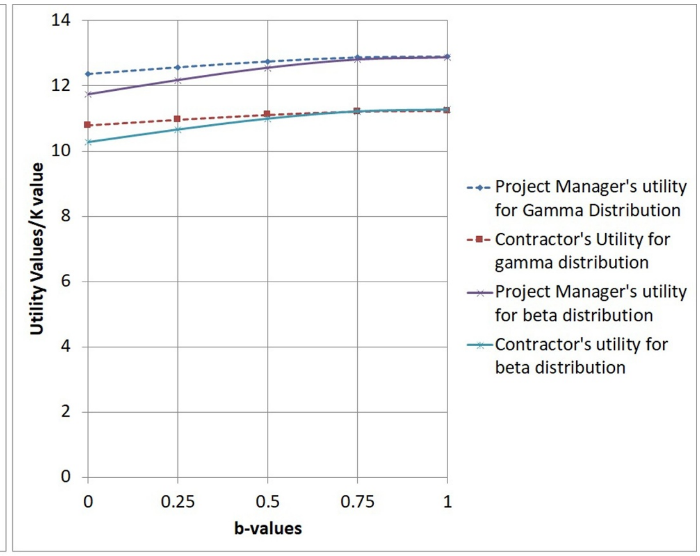
Secondly, three bargaining models were analyzed: Nash, Kalai-Smorodinsky and Utilitarian. The results from the analyzes of these three also revealed further implications for practice. In most of the cases, the cost-plus contract was the best linear contract, i.e. $b = 1$. However, in the Utilitarian case, the contractor's utility U_{co} was the same for $b \in [0, 1]$. Hence the contractor is insensitive to the value of b. This has the problem that if the contractor is slightly less risk-averse than the project manager believes or if the contractor has a secondary objective of maximizing his profit, then the contractor will prefer the utility they receive if b is fixed at zero and then the value of a is chosen, by the bargaining model, rather if b is fixed at one before choosing a. For the Nash and Kalai-Smorodinsky bargaining models, the contractor's utility was higher at $b = 1$ than for other values of b, and so they do not suffer from this instability. Consequently, this highlights a significant problem with the Utilitarian bargaining model. This may be due to the fact that the utilitarian approach is one of the extreme bargaining approaches.

Although cost-plus contracts solve the bargaining models, and so provide the optimal solution, they have the drawback as far as project managers are concerned, that the project manager lacks direct control of the costs that the contractor incurs. Consequently, there can be a concern that the contractor is making expenditure decisions that may be higher than the optimal on the grounds that they will be reimbursed for the project costs. This fear of being exploited may lead to some trade-off in the decision making of the project manager (between the loss of utility for not selecting a cost-plus contract and the loss of control due to the opportunistic behavior of the contractor) of the project manager. In practice, if the project manager has less trust over the contractor's opportunistic behavior, she may choose a value of b that is below 1 if the utility loss for not selecting the optimal is not significant. Despite these challenges, it is used in practice. We found evidence of its use from a report of the European Union on the projects: treatment, valorisation and solid-waste disposal of inter-municipal system of the “Litoral Centroâg region (ERSUC), and Sanitation sub-systems of Barreiro/Moita and Seixal (SIMARSUL) in Portugal [9]. We also found recent evidence of the use of cost-based contracts including cost-plus contracts, e.g. the USA's Department of Defence, recently awarded a cost-plus fixed-fee contract to Science Application International Corporation [22]. Bargaining power can play an important role in the selection of the type of the contract and a wrong selection could make things less profitable leading to organizations involved going into administration. This is quite common in construction sector projects such as recent failures in the UK of JistCourt [6] and Carillion [18]. The main reason cited for these failures was the presence of less profitable projects. In fact, Goodley [18] reported that the failure of Carillion was due to a popular phenomenon “subbie-bashing” where the small and medium firms are financially exploited by powerful large farms in their supply chain. Above all, the value of the current research is in proving that the best solution occurs at $b = 1$, and so allows the loss of utility from choosing a non-optimal value of b to be determined.

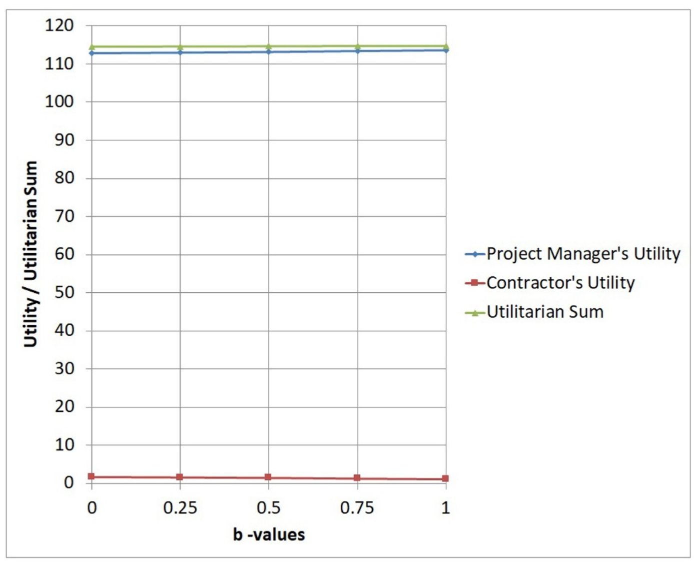
Furthermore, our sensitivity analysis investigated how things changed when the project manager also became risk-averse. This has several practical implications. In reality, it is likely that the project manager may have some degree of risk-aversion in many cases. It can be easily shown (both analytically and numerically) that the risk-averse project manager's best interest was to offer a fixed price contract to avoid any exploitation of uncontrolled costs being passed over to her by the contractor as discussed in the last paragraph. On the contrary, our analysis showed that the risk-averse contractor was inclined to bargain for a cost-plus contract as he was having the fear of uncertainty in cost estimation at the beginning of the project. Depending on the degree of risk-aversion of the member, the optimal results were neither a fixed price ($b = 0$) nor a pure cost-plus ($b = 1$), but rather a cost-sharing contract with $0 < b < 1$. We also found when both are more or less equally risk-averse, the optimal solution tends to be around $b = 0.5$ i.e. the cost incurred by the contractor is equally shared between the project manager and the contractor. In reality, the bargaining power of the members of the supply chain also plays an important role in this type of bargaining where both members are risk-averse. As a result, things may not always be optimal and this may become detrimental for the member with less bargaining power.



(a) For Nash bargaining



(b) For Kalai-Smorodinsky bargaining



(c) For Utilitarian bargaining

Fig. 6. Results for decreasing absolute risk-aversion.

Further sensitivity analysis with a different type of risk-aversion (Decreasing Absolute Risk Aversion) showed how the efficiency of the cost-plus contracts may change (decrease in this case) when the type of risk aversion changes. The main reason for this change was the change in the contractor's level of financial engagement (more in this case and as a result expecting less risk premium). Again, the results were quite similar for the Nash and Kalai-Smorodisnky bargaining cases, but somewhat different for the Utilitarian bargaining cases. This may be again due to the extreme bargaining nature of this type of utilitarian approach.

There are certain areas where our models can be extended for future research. Firstly, how the optimal solutions may change in the presence of another third party such as a mediator can be investigated in future work (as highlighted in a different context in the recent work of Fairchild [12]). Furthermore, we showed numerically with multiple scenarios of how the optimal contract was neither a fixed price ($b = 0$) nor a pure cost-plus ($b = 1$) when both the members were risk-averse. Further research can be conducted with the empirical validation of this type of situation with how the bargaining power plays a role in it. The proposed models of this paper could also be extended to the situation where several potential contractors are competing through an auction process. Of particular interest would be how a fixed b, e.g. $b = 0.5$, interacts with the risk aversion level of the bidder, and so, for example, quantifying how much smaller contractors are disadvantaged when b is small.

CRediT authorship contribution statement

Niladri Palit: Conceptualization, Methodology, Formal analysis, Writing - original draft. **Andrew Brint:** Supervision, Writing - review & editing, Project administration.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Proof of Lemma 3 and Proposition 1

Differentiating equation (9) with respect to b, the following is derived

$$\frac{dW}{db} = \begin{cases} -\dfrac{W\eta\phi\omega}{\{1-\eta\phi(1-b)\}} & \text{gamma dist cost} \\ -\displaystyle\sum_{i=1}^{\infty}\left\{\prod_{r=0}^{i-1}\left(\frac{c+r}{c+d+r}\right)\right\}\frac{\phi^i i\eta^i(1-b)^{i-1}}{i!} & \text{beta dist cost} \end{cases} \tag{A.1}$$

As mentioned in the problem description i.e in Section 3, $\eta > 0$ for the risk-averse members. It can also be shown that $q - a_0 > 0$, otherwise the project manager's utility would be negative and she would never participate in the bargaining. Thus, from equation (12), it can be shown that $\frac{1}{(1-\eta\phi)^{\omega}} > 0$ for a gamma distributed cost when η and ϕ both are positive. Thus, $\eta\phi < 1$. It was shown on page 9, Section 4 that W = 1 for $b = 1$ and $W = W_0$ for $b = 0$. It is also assumed before that $0 \le b \le 1$. This leads to $0 \le \eta\phi(1-b) < 1$ and $1 - \eta\phi(1-b) > 0$. Thus, the value of $\frac{dW}{db}$ is negative. This means W is a decreasing function of b for $b \in [0, 1]$. Since W is decreasing in b for $b \in [0, 1]$, W is positive as the minimum value could be 1.

Now using the value of $\mu = \omega\phi$ from equation (11) and the value of $\frac{dW}{db}$ from Eq. (A.1) in equation (19), the value of $-\frac{da}{db} - \mu$ for the gamma distributed cost in equation (21) is derived.

In the equation (21), the right-hand side of the equation is positive with $0 \le b < 1$ and zero with b = 1. Thus, the value of the term $(-\frac{da}{db} - \mu)$ is: positive with $0 \le b < 1$; and zero with b = 1 for a gamma distributed cost function X. Using this observation in equation (15), U_{pm} is found to be increasing in b in the range $0 \le b < 1$ and attains the maximum with b = 1. It is also assumed earlier that $\eta > 0$. Thus, from equation (17), $-\frac{dA}{db} > 0$. This proves that $\frac{dU_{co}}{db} \ge 0$ for $b \in [0, 1]$. Thus, U_{co} is increasing in b with $0 \le b < 1$ and attains the maximum with $b = 1$. This proves the case for gamma distributed cost for Lemma 3. Using these above observations in equation (14), it can be shown that $\frac{dN}{db} > 0$ for $0 \le b < 1$ and $\frac{dN}{db} = 0$ for $b = 1$. This proves the case for Nash product with gamma distributed cost in lemma 3.

Now for the beta distributed cost, expanding the right-hand side of Eq. (A.1), the following is derived

$$\begin{aligned}\frac{dW}{db} &= -\left(\frac{\eta\phi c}{c+d}\right) - \left(\frac{2\eta^2(1-b)\phi^2 c(c+1)}{2!(c+d)(c+d+1)}\right) - \left(\frac{3\eta^3(1-b)^2\phi^3 c(c+1)(c+2)}{3!(c+d)(c+d+1)(c+d+2)}\right) - \cdots \\ &= -\left(\frac{\eta\phi c}{c+d}\right)\left[1 + \left(\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}\right) + \left(\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}\right) + \cdots\right]\end{aligned} \tag{A.2}$$

Using this value of $\frac{dW}{db}$ and the $\mu = \frac{\phi c}{c+d}$ for the beta distributed cost in Eq. (19) and expanding the value of W from Eq. (9), we get the following

$$\begin{aligned}-\frac{da}{db} - \mu &= -\frac{1}{A}\left[\frac{1}{\eta W}\left(\frac{dW}{db}\right) + \mu\right] \\ &= \frac{\left(\frac{\eta\phi c}{c+d}\right)\left[1 + \left(\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}\right) + \left(\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}\right) + \cdots\right]}{\eta A\left[1 + \eta(1-b)\left\{\frac{\phi c}{c+d}\right\} + \frac{\eta^2(1-b)^2}{2!}\left\{\frac{\phi^2 c(c+1)}{(c+d)(c+d+1)}\right\} + \cdots\right]} - \frac{1}{A}\left(\frac{\phi c}{c+d}\right) \\ &= \frac{\eta(1-b)\phi^2}{A}\left(\frac{c}{c+d}\right)\left[\frac{\left(\frac{d}{(c+d)(c+d+1)}\right) + \frac{\eta(1-b)\phi}{2!}\left(\frac{2d(c+1)}{(c+d)(c+d+1)(c+d+2)}\right) + \cdots}{1 + \eta(1-b)\left[\frac{\phi c}{c+d}\right] + \frac{\eta^2(1-b)^2}{2!}\left[\frac{\phi^2 c(c+1)}{(c+d)(c+d+1)}\right] + \cdots}\right]\end{aligned}$$

This above equation serves the case for the beta distributed cost in equation (21). It can be easily shown that the term $1 + \eta(1-b)\left[\frac{\phi c}{c+d}\right] + \frac{\eta^2(1-b)^2}{2!}\left[\frac{\phi^2 c(c+1)}{(c+d)(c+d+1)}\right] + \cdots$, in the above equation is W. Similar to the argument for the case of gamma distributed cost function, it can be shown that W is positive. The parameters η, c, and d are all positive. It is also shown that A is positive. Thus, when $0 \le b < 1$, the right-hand side of the equation (21) becomes positive. When $b = 1$, the right hand side is zero. Hence, $-\frac{da}{db} - \mu \ge 0$ with $0 \le b \le 1$. This again concludes that U_{pm} is increasing in b with $0 \le b < 1$ and attains the maximum value with b = 1. Using the observations above in equation (17), it can be shown $-\frac{dA}{db} \ge 0$ for $b \in [0, 1]$ i.e. $\frac{dU_{co}}{db} \ge 0$ for $b \in [0, 1]$. Thus, similar to the calculations of the gamma distribution, it can be shown U_{co} is increasing in b for $0 \le b < 1$ and attains the maximum at b = 1. This proves the case for beta distributed cost in lemma 3. Using these observations in equation (14), it can be shown that $\frac{dN}{db} > 0$ for $0 \le b < 1$ and $\frac{dN}{db} = 0$ for $b = 1$. In other words, N is increasing in b for $0 \le b < 1$ and is maximum at b = 1. This proves the case for Nash product with beta distributed cost in lemma 3. The proposition 1 is the generalization following from lemma 3 and can be very easily proved for other non-normal continuous distributions. The paper by Lippman et al. [33] has derived models for the normal distributed cost and Nash bargaining.

Appendix B. Proof of Lemma 6 and Proposition 2

Using the value of W (for the gamma distributed cost function) from equation (9) of Section 4 and $\frac{dW}{db}$ (for the gamma distributed cost function) from Eq. (A.1) in Eq. (31) in Section 5 and rearranging the values in terms of W, we get the right-hand side of the equation (32) in Section 5 for the gamma distributed cost function. It was shown Appendix A that $1-\eta(1-b)\phi>0$ for a gamma distributed cost function with shape parameter ω and scale parameter ϕ. Hence, the right-hand side of the equation (32) is positive for $0 \leq b < 1$ and zero for $b=1$ for a gamma distributed cost function with the shape parameter ω and the scale parameter ϕ. Hence, combining this with the findings from equation (15), it can be said that U_{pm} is increasing in $0 \leq b < 1$ and attains the maximum value at $b=1$. By replacing the value of $\frac{dU_{co}}{db}$ on the right-hand side of equation (30) in Section 5 and rearranging the values, it can be easily shown that $(-\frac{da}{db}-\mu)=B\frac{dU_{co}}{db}$. Now the term B is positive as argued in Section 5. Hence, from the above observation, if $(-\frac{da}{db}-\mu)$ is positive for $b\in[0,1)$ and zero for $b=1$, then same is the case for $\frac{dU_{co}}{db}$. This means U_{co} is increasing in b for $0 \leq b < 1$ and attains the maximum value at $b=1$. These observations about U_{pm} and U_{co} prove the arguments presented in lemma 6 for the gamma distributed cost.

For a beta distributed cost X with scale parameter ϕ $(0 < X < \phi)$ and shape parameters c and d, using the value of $\frac{dW}{db}$ from Eq. (A.2) and the expanded form of W from equation (9) in equation (31) gives

$$\left(-\frac{da}{db}-\mu\right)(1+B\eta e^{-\eta a}W)$$
$$=Be^{-\eta a}\left[\frac{\eta\phi c}{c+d}\left\{1+\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}+\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}+\cdots\right\}\right]$$
$$-B\eta e^{-\eta a}\mu\left[1+\frac{\eta\phi(1-b)c}{(c+d)}+\frac{\eta^2\phi^2(1-b)^2c(c+1)}{2!(c+d)(c+d+1)}+\cdots\right]$$
$$=Be^{-\eta a}\eta\mu\left[\eta(1-b)\phi\left\{\frac{(c+1)(c+d)-c(c+d+1)}{(c+d)(c+d+1)}\right\}\right]$$
$$+Be^{-\eta a}\eta\mu\left[\frac{\eta^2(1-b)^2\phi^2}{2!}\left\{\frac{(c+1)(c+2)(c+d)-c(c+1)(c+d+2)}{(c+d)(c+d+1)(c+d+2)}\right\}\right]+\cdots$$
$$=Be^{-\eta a}\eta\mu\left[\eta(1-b)\phi\left\{\frac{d}{(c+d)(c+d+1)}\right\}+\frac{\eta^2(1-b)^2\phi^2}{2!}\left\{\frac{(c+1)2d}{(c+d)(c+d+1)(c+d+2)}\right\}+\cdots\right]$$

[The value $\frac{\phi c}{c+d}$ was replaced with μ for the beta distributed cost in the above derivation on the right hand side in step 2]

From the above equation, it can be shown that the right-hand side of the equation is positive for $0 \leq b < 1$ and zero for $b=1$. This is because η is positive (as argued earlier), and c and d are both assumed as positive as well. Following the steps shown in gamma distributed cost, it can be again shown that U_{pm} and U_{co} both are increasing in b for $0 \leq b < 1$ and maximum at b = 1. These observations about U_{pm} and U_{co} prove the arguments presented in lemma 6 for the beta distributed cost.

Now using these above observations for U_{pm} and U_{co} (for gamma and beta distributed costs) with what was found in equation (27), it can be shown that the Kalai-Smorodinsky value K value is increasing in b for $0 \leq b < 1$ and is maximum for $b=1$. This further proves the argument about the Kalai–Smorodinsky value K in lemma 6. The proposition 2 is the generalization following from lemma 6 and can be very easily proved for other non-normal continuous distributions. For the normally distributed cost, the standard deviation and the mean are not the functions of the same combination of parameters. Hence, it would be interesting to see this in future work if the derived models can be generalized for the case of normal distributed costs.

Appendix C. Proof for Lemma 7 and Proposition 3

Using the values of W from equation (9) and $\frac{dW}{db}$ from Eq. (A.1) for the gamma distribution in equation (37) gives

$$\frac{da}{db}=-\frac{\phi\omega}{1-\eta(1-b)\phi} \tag{C.1}$$

Using the value of $\mu=\omega\phi$ from Eq. (11), and using the value of $\frac{da}{db}$ from the Eq. (C.1) in Eq. (37) or the values of W from Eq. (9) and the value of $\frac{dW}{db}$ from Eq. (A.2), in Eq. (38), we get the right-hand side of the Eq.(39) for the gamma distributed cost. Following the arguments from Appendix A, it can be shown that $\eta\phi < 1$, and $1-\eta(1-b)\phi>0$. Hence, it can be shown that the right-hand side of the Eq. (39) for the gamma distributed cost is positive for $0 \leq b < 1$ and zero for $b=1$. Combining this observation with the findings from Eq. (15), U_{pm} is found to be increasing in b for $0 \leq b < 1$ and attains the maximum with b = 1. From Eqs. (15), (37) and (38), it can be easily shown that $\frac{dU(z,D)}{db}=\frac{dU_{pm}}{db}$. Now differentiating both sides of the Eq. (34), and replacing the value of $\frac{dU_{pm}}{db}$, it can be easily found that $\frac{dU_{co}}{db}=0$. Hence, U_{co} remains unchanged for $b\in[0,1]$ and U(z,D) changes the same way U_{pm} does. This proves the case for gamma distributed cost as argued in lemma 7.

Using these values of W for the beta distribution from Eq. (9) and $\frac{dW}{db}$ from Eq. (A.2) in Eqs. (37) and (36), we get

$$\frac{da}{db}=-\frac{\mu}{W}\left\{1+\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}+\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}+\cdots\right\} \tag{C.2}$$

[The mean value of beta distributed cost $\mu=\frac{\phi c}{c+d}$] and

$$\frac{dU(z,D)}{db}$$

$$= -\frac{\mu}{W}\left\{1+\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}+\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}+\cdots\right\}[\eta e^{-\eta a}W-1]$$

$$-\mu+e^{-\eta a}\left[\eta\mu\left\{1+\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}+\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}+\cdots\right\}\right]$$

$$=\frac{\mu\left[\left\{1+\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}+\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}+\cdots\right\}-W\right]}{W}$$

Expanding the value W for the beta distributed cost from Eq. (9)

$$=\frac{\mu\left[\left\{1+\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}+\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}+\cdots\right\}-\left\{1+\frac{\eta\phi(1-b)c}{c+d}+\frac{\eta^2\phi^2(1-b)^2c(c+1)}{2!(c+d)(c+d+1)}+\cdots\right\}\right]}{1+\frac{\eta\phi(1-b)c}{c+d}+\frac{\eta^2\phi^2(1-b)^2c(c+1)}{2!(c+d)(c+d+1)}+\cdots}$$

[Converting the denominator in the last step back to W]

$$=\frac{\mu}{W}\left[\eta(1-b)\phi\left\{\frac{d}{(c+d)(c+d+1)}\right\}+\frac{\eta^2(1-b)^2\phi^2}{2!}\left\{\frac{(c+1)2d}{(c+d)(c+d+1)(c+d+2)}\right\}+\cdots\right]$$

Following the arguments from Appendix B,it can be shown that the right-hand side of the above equation is positive for $0 \le b < 1$ and zero for b = 1. This leads to the conclusion that $U(z, D)$ is increasing in b for $0 \le b < 1$ and reaches a maximum at b = 1. Following the same steps for the case of gamma distributed cost with utilitarian bargaining, it can be shown that $\frac{dU_{co}}{db} = 0$ and $\frac{dU_{pm}}{db} = \frac{dU(z,D)}{db}$. Hence, U_{co} remains unchanged for $b \in [0, 1]$ and U(z,D) and U_{pm} are found to be increasing in b for $0 \le b < 1$ and attains the maximum with b = 1. This proves the case for beta distributed cost as argued in lemma 7. The proposition 3 is the generalization following from lemma 7 and can be very easily proved for other non-normal continuous distributions. For the normal distributed cost, the standard deviation and the mean are not the functions of the same combination of parameters. Hence, it would be interesting to see this in future work if the derived models can be generalized for the case of normally distributed costs.

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